Level7opaedia

'A level is a level'

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Please note that Using and Applying assessment criteria are not included within the Levelopaedia

Numbers and the Number System

Understand and use proportionality	
A house was worth £215,000 at the start of 2004, and over the course of the next year its value rose by 13.5%. Using only multiplication, work out the value of the house at the end of the year. What happens if you increase an amount by 100%, then decrease the result by 100%? If you increase an amount by 10%, then decrease the new amount by 10%, what is the percentage change?	(Using a proportional reasoning table) show me one way to solve this problem. And another... True/Never/Sometimes: Increasing an amount by 10% then decreasing the answer by 10% takes you back to where you started Convince me that decreasing an amount by 10% then decreasing the answer by 10% leaves you with 81% of the amount you started with

Calculating

Calculate the result of any proportional change using multiplicative methods	
A house was worth £215,000 at the start of 2004, and over the course of the next year its value rose by 13.5%. Work out the value of the house at the end of the year. A car was bought for £17,500, but depreciated by 27% over the first year. Work out the value of the car at the end of the year. Is a 50% increase followed by a 50% increase the same as doubling? Explain your answer.	What is wrong: To increase an amount by 50% you multiply by 0.5 I want to reduce an amount by 70%, which of the following numbers could I multiply by to help me to find the answer, and which one is the odd one out: 0.3, 1.3, 0.7 True/Never/Sometimes: When you want to increase an amount by a percentage, you always multiply by a number above 1. Convince me that ▪ Your answer is reasonable by using another method ▪ The graph of y = 1.2x can help to find a 20% increase of any amount.
Understand the effects of multiplying and dividing by numbers between 0 and 1	
'Multiplication always makes things bigger' - True or false? Explain your answer. Give three division calculations where the answer is bigger than the starting number	Show me two numbers, one of which is between 0 and 1, which multiply to give 50. Repeat for division. Find me a number you can divide by to achieve an increase of 100% What is the same/different about: ▪ 0.8 ÷ 0.1 ▪ 16 × 0.5 ▪ 1.6 ÷ 0.5 ▪ 1.6 ÷ 0.2 True/Never/Sometimes: Multiplication always makes things bigger Convince me that division can make the answer bigger.
Add, subtract, multiply and divide fractions	
Find the area and perimeter of a rectangle measuring 4¾ inches by $6^3/_8$ inches. Using 22/7 as an approximation for pi, estimate the area of a circle with diameter 28mm.	Show me an example of: ▪ Two fractions that add together to make a whole. ▪ Three fractions that add together to make a half. ▪ A fraction that is bigger than 5 ▪ A fraction that is between 3 and 4 ▪ Two fractions that multiply to make a whole What is the same/different about ▪ 2/5 × 2 ▪ 3/4 ÷ 2/5 ▪ 1/3 + 1/9 ▪ 6/7 – 2/7 True/Never/Sometimes: Multiplying two fractions together gives a bigger answer than either of the fractions you are multiplying Convince me that: ▪ Any two fractions can be added or subtracted. ▪ Any number of fractions can be added or subtracted from each other

Make and justify estimates and approximations of calculations; estimate calculations by rounding numbers to one significant figure and multiplying and dividing mentally	
(186.3 x 88.6)/(27.2 x 22.8)	Show me an example of a division calculation using decimals that approximates to 60 True/Never/Sometimes: ▪ Rounding numbers to the nearest integer is a good way to find an estimate in a calculation ▪ Estimates give an over estimate if you round the numbers up Convince me that your estimate gives a good approximation
Use a calculator efficiently and appropriately to perform complex calculations with numbers of any size, knowing not to round during intermediate steps of a calculation	
Finding areas of shapes created by parts of a circle	Show me an example of: ▪ A calculation that gives the same answer with or without brackets ▪ A calculation that gives a different answer with and without brackets What is the same/different about: ▪ (3.9 – 4.5)/ 2.5 ▪ (5 × 3)/(0.4 × 0.2) ▪ 1.805/(1.0 - 1.3) ▪ ($^-$5.3)^2 Convince me that you cannot do some calculations without brackets if you put your working into the calculator in one go

Algebra

Square a linear expression, and expand and simplify the product of two linear expressions of the form $(x \pm n)$ and simplify the corresponding quadratic expression	
Multiply out these brackets and simplify the result: (g+4)(g-3)	Show me an expression in the form (x + a)(x + b) which when expanded (i) the x coefficient is equal to the constant term (ii) the x coefficient is greater than the constant term True/Never/Sometimes: $x^2 + 2x + 4 = (x+1)(x+a)$ Convince me that $x^2 + 2ax + a^2 = (x+a)(x+a)$
Use algebraic and graphical methods to solve simultaneous linear equations in two variables	
If 4x+y=27 and x=2y, find the value of x and y using an algebraic method Solve these simultaneous equations using an algebraic method: 3a+2b=16, 5a-b=18 Solve graphically the simultaneous equations x+3y=11 and 5x-2y=4	Show me a point that lies on the line 2x + 3y = 12 Show me the equations of two lines which (i) intersect, (ii) are parallel Show me two lines that intersect in the 1st, 2nd, 3rd etc. quadrant True/Never/Sometimes: If you multiply 2x – 3y = 3 by a constant you get the same equation in a different format. Convince me that 2x + 3y = 4 and 6y = 4x – 6 has only one solution
Solve inequalities in one variable and represent the solution set on a number line	
Solve 3(j+4) > 24 showing the solution on a number line	Show me an example of x where 2x + 3 < 4 Show me an example of a which makes ax > b the same as x<b/a What is the same/different about these two inequalities x + 4 < 4 and x + 4 > 6 and these two inequalities x + 4 > 4 and x + 4 < 6 How can you change 5 – 3x < 14 so x is the subject of the inequality? True/Never/Sometimes: ax > b and x < b/a
Use formulae from mathematics and other subjects; substitute numbers into expressions and formulae; derive a formula and, in simple cases, change its subject	
See the full range of examples on pages 138-141 of the KS3 Framework supplement of examples	Show me an example of a formula that has the value 7 when a = 2 and b = 3 What is wrong: 5P + 3Q = 12 is the same as 10P + 7Q = 24 How can you change v = u + at so that (i) u is the subject (ii) t is the subject? Convince me that y =– 3 when p = 15 and x = 2 for the formula p = 2x – 3y
Find the next term and nth term of quadratic sequences and functions and explore their properties	
-3, 0, 5, 12, 21, … 1/2, 1/6, 1/12, 1/20, 1/30, ...	Show me an example of a sequence with a quadratic nth term How can you continue a sequence starting 1, 2, ... so that it has a quadratic nth term? How can you change 3, 6, 9, 18, 27 so it becomes the first five terms of a quadratic sequence?

	What is the same different about the sequences: ▪ 1, 4, 9, 16, 25 ▪ 0, 3, 8, 15, 24 ▪ 4, 7, 12, 19, 28 ▪ 2, 8, 18, 32, 50 True/Never/Sometimes: ▪ Sequences with an equivalent second difference have a quadratic nth term ▪ Sequences with an unequal first difference pattern have a quadratic nth term ▪ The second difference for a quadratic sequence is always 2 Convince me that 5, 9, 15, 23, 33, ... has a quadratic nth term
Plot graphs of simple quadratic and cubic functions	
▪ $y = x^2$ ▪ $y = 3x^2 + 4$ ▪ $y = x^3$	Show me an example of (i) a quadratic graph, (ii) a cubic graph Show me a coordinate that lies on the graph ▪ $y=2x^2-2$ ▪ $y=x^3$ How can you change the following coordinates so that when plotted they lie on a quadratic graph: (-2, -14), (-1, -5), (0, 2), (1, 5), (2, 14) Convince me that there are no coordinates on the graph of $y=3x^2+4$ which lie below the x-axis

Shape, Space and Measures

Understand and apply Pythagoras' theorem when solving problems in 2-D	
Find a missing hypotenuse in a right-angled triangle Find a missing shorter side in a right-angled triangle Identify triangles that must be right-angled from their side-lengths	Show me and example of: ▪ A Pythagorean triple ▪ A problem which can be solved using Pythagoras' Theorem What is the same/different about (Diagram of) a triangle with sides 5cm, 12cm and an unknown hypotenuse and (diagram of) a triangle with sides 5cm, 12cm and an unknown shorter side True/Never/Sometimes: Pythagoras' Theorem can be used to find the lengths of sides in triangles
Calculate lengths, areas and volumes in plane shapes and right prisms	
See the full range of examples on pages 236-241 of the KS3 Framework supplement of examples. e.g. ▪ 'box of coffee' (p.241) ▪ find the height of a cylinder given its volume and cross-sectional area	Show me and example of a: ▪ cuboid/cylinder/triangular prism with volume 24 ▪ cuboid with a surface area of 24 What is the same/different about a cuboid with dimensions 3, 4, 2 and a cuboid with dimensions 1, 3 , 8 True/Never/Sometimes: ▪ Cuboids with the same volume have the same surface area ▪ A cylinder can never have the same volume as a cuboid Convince me that you can find the volume of a hexagonal prism
Enlarge 2-D shapes, given a centre of enlargement and a fractional scale factor, on paper and using ICT; recognise the similarity of the resulting shapes	
Investigate the standard paper sizes A1, A2, A3, … Use 'Geomat' or 'Autograph' to carry out enlargements Decide whether two triangles are similar in simple cases	Show me and example of: ▪ An enlargement with a fractional scale factor ▪ An enlargement with a negative scale factor What is the same/different: An enlargement of a triangle where the centre of the enlargement is inside, on the perimeter, or outside the original triangle True/Never/Sometimes: ▪ An enlargement always produces a larger shape ▪ An enlargement always produces an image on the same side of the centre as the object ▪ An image is never the same size as an object ▪ An image is never congruent to the image ▪ A3 paper is an enlargement of A1 paper Convince me that: ▪ any A sized paper is an enlargement of any other A sized paper ▪ the ratio of the sides of any A sized paper is the square root of 2
Find the locus of a point that moves according to a given rule, both by reasoning and using ICT.	
Visualise the result of spinning 2D shapes around a line (see p.225 of the KS3 Framework supplement of examples) Trace the path of a vertex of a square as it is toppled Practical applications of the perpendicular bisector	Show me and example of: ▪ A point equidistant from these two points, and another and another …. ▪ A point a metre from this line, and another, and another… ▪ A point a metre from this point, and another,… ▪ The locus of the path traced out by the centres of circles which have two given lines as

<table>
<tr><td></td><td>tangents

What is the same/different about the path traced out by the centre of a circle being rolled along a straight line and the centre of a square being rolled along a straight line

Convince me that
▪ You can trace out the path traced out by the tip of a windscreen wiper
▪ The angle in a semicircle is always 90 degrees</td></tr>
<tr><td colspan="2">Recognise that measurements given to the nearest whole unit may be inaccurate by up to one half of the unit in either direction</td></tr>
<tr><td>Understand upper and lower bounds

Find maximum and minimum values for an measurement that has been rounded to a given degree of accuracy

Use inequality symbols in this context</td><td>Show me and example of a number which is 0.6 when rounded to 1 decimal place, ... and then is 0.60 when rounded to 2 decimal places, ... and then is 0.600 when rounded to three decimal places.

What is the same/different about $- 3 \le x < 4$ and $3 < x < 4$

True/Never/Sometimes:
▪ I would pick a runner whose time for running 100m is recorded as 13.3 seconds rather than 13. 30 seconds
▪ If the weight limit in the lift is 660kg and each of 6 people weigh 110kg they will all fit in the lift.

Convince me that between two numbers you can always find another number (think about 0.9 recurring and 1!)</td></tr>
<tr><td colspan="2">Understand and use measures of speed (and other compound measures such as density or pressure) to solve problems</td></tr>
<tr><td>Use compound measures in science, geography or PE

Compare the speed of a sprinter (100m in 10 seconds) to the speed of a cyclist (13 miles in 1 hour)</td><td>Show me and example of a suitable unit for the measurement of the speed of a boat, an aeroplane, the space shuttle, a snail, a Year 11 walking to my lesson, …

What is the same/different:
▪ 5 mph and 8km per hour
▪ A distance-time graph with a positive gradient and a distance-time graph with a negative gradient

True/Never/Sometimes: A sprinter travelling 100m in 10 seconds is faster than a cyclist travelling 13 miles in 1 hour

Convince me that:
▪ You need to put time on the horizontal axis
▪ The area under a velocity-time graph gives you the distance travelled</td></tr>
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Handling Data

Suggest a problem to explore using statistical methods, frame questions and raise conjectures; identify possible sources of bias and plan how to minimise it	
See the range of examples on page 249 of the KS3 Framework supplement of examples e.g. ▪ Are development indicators (for countries) consistent with each other? ▪ How available are fairly-traded goods in local shops?	Show me an example of a situation in which biased data would result What is the same/different about ▪ Using randomly generated mobile telephone numbers to contact people and using randomly generated landline telephone numbers ▪ Posting a questionnaire to households and posing a questionnaire to individuals by interviewing them True/Never/Sometimes: Data is always biased Convince me that it is possible to explore this situation / hypothesis
Select, construct and modify, on paper and using ICT, suitable graphical representation to progress an enquiry, including frequency polygons and lines of best fit on scatter graphs	
Use superimposed frequency polygons in preference to bar charts When plotting a line of best fit, find the mean point (x,y) Make a prediction using a line of best fit Recognise the fact that the prediction is subject to error Recognise the fact that the line of best fit should not pass beyond the range of known values	Show me an example of a graph which represents this data clearly What is the same/different about: ▪ the (two or more different types of) graphs that represent this (one set of) data ▪ these sets of data (which contain some discrete/continuous/bivariate data/not bivariate) True/Never/Sometimes: ▪ a scatter diagram should be used to represent bivariate data ▪ the easier it is to place a line of best fit on a scatter diagram, the stronger the correlation displayed ▪ the scales on a graph representing grouped data should be labelled with the endpoints of the groups Convince me that this is the most appropriate graph to use in this case
Estimate the mean, median and range of a set of grouped data and determine the modal class, selecting the statistic most appropriate to the line of enquiry	
Recognise the meaning of 'calculate an estimate' when estimating the mean Estimate the median and range from a grouped frequency table Estimate the mean from a grouped frequency diagram	Show me an example of a set of grouped data with an estimated: ▪ range of 35 ▪ median of 22.5 ▪ median of 22.5 and range of 35 ▪ mean of 7.4 (to 1dp) What is the same about/different about 'estimate the mean of...' and 'calculate an estimate of the mean of...'? True/Never/Sometimes: ▪ The estimated mean always gives the best estimate for set of grouped data ▪ The best estimate for the range is always the largest possible value minus the smallest possible value ▪ The median lies in the central group Convince me that the estimated mean is not always

	the most appropriate average
Compare two or more distributions and make inferences, using the shape of the distributions and measures of average and range	
See the range of examples on page 273 of the KS3 Framework supplement of examples e.g. ▪ compare athletic performances in different year groups ▪ compare populations	Show me an example of: Two sets of data with the same mean, where one set has the mean = median, and the other set has mean > median. What might this data look like if represented graphically? What is the same about/different about the two sets of data 7, 10, 8, 7, 4, 13, 9 and 7, 9, 3, 11, 9, 2, 6 True/Never/Sometimes: ▪ mean > median ▪ range = Mean ▪ range = median ▪ mean < median ▪ mean > median > range ▪ mean < range < median Convince me that (given two sets of data/graphs) you would choose to 'buy brand A' instead of 'brand B'
Understand relative frequency as an estimate of probability and use this to compare outcomes of an experiment	
Recognise that repeated trials result in experimental probability tending to a limit, and that this limit may be the only way to estimate probability	Show me an example of: a situation that would require the use of experimenting to estimate a probability What is the same about/different about using theoretical probability to find the probability of obtaining a 6 when you roll a dice, and using experimental probability for the same purpose True/Never/Sometimes: ▪ Experimental probabliity is more reliable than theoretical probability ▪ Experimental probabliity gets closer to the true probability as more trials are carried out ▪ Relative frequency finds the true probability Convince me that experimental probabliity is more reliable than theoretical probability
Examine critically the results of a statistical enquiry, and justify the choice of statistical representation in written presentations	
Examine data for cause and effect Try to explain anomalies (see example on p.271 of KS3 Framework supplement of examples - engine size / acceleration) Recognise that establishing a correlation or connection in statistical situations does not necessarily imply that one variable causes change to another, and that there may be external factors affecting both.	Show me an example of a situation where there is positive correlation, but it is unlikely that there is causation What is the same/different about the statements 'my hypothesis is true' and 'there is strong evidence to support my hypothesis' True/Never/Sometimes: ▪ Your hypothesis is true if you have found enough evidence to support it ▪ You have failed if your hypothesis appears to be flawed Convince me that ▪ this is the most appropriate graph to use in this case ▪ there is evidence to support your hypothesis