Level8opaedia

'A level is a level'

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Please note that Using and Applying assessment criteria are not included within the Levelopaedia

Numbers and the Number System

Understand the equivalence between recurring decimals and fractions	
Decide which of the following fractions are equivalent to terminating decimals: 3/5, 3/11, 7/30, 9/22, 9/20 Write 0.45454545… as a fraction in its simplest terms	Show me an example of: ▪ A fraction which terminates when written as a decimal ▪ A fraction which has a recurring decimal equivalent with two different digits repeating What is the same about/different about 13/33, 44/333 and 7/40 True/Never/Sometimes: Fractions with a denominator which has a factor of 2 terminate when written as a decimal Convince me that 0.417417417… = 139/333

Calculating

Use fractions or percentages to solve problems involving repeated proportional changes or the calculation of the original quantity given the result of a proportional change	
Calculations involving compound interest or population growth Use of 'proportional reasoning tables' to calculate the original amount	Show me an example of a problem involving repeated percentage change What is the same about/different about: ▪ £130 x 1.09 x 1.09 ▪ £130 x $(1.09)^2$ ▪ (£130 x 0.09 + £130) x 0.09 + (£130 x 0.09 + £130) ▪ £150 x 0.85 x 0.85 Convince me that using powers is the most efficient way of solving this problem
Solve problems involving calculating with powers, roots and numbers expressed in standard form, checking for correct order of magnitude and using a calculator as appropriate	
Knowledge and use of laws of indices for multiplication and division Conversion between 'ordinary form' and standard form Knowledge and use of the standard form function of a scientific calculator	Show me an example of ▪ Two calculations using powers that give the same value. ▪ Two calculations using roots that give the same value What is the same/different about: ▪ 1.1^2 ▪ 1.2 × 10-3 ▪ $\sqrt{(1.2/ (5/6))}$ ▪ 0.45 ▪ $\sqrt[3]{0.009261}$ True/Sometimes/Never: ▪ Cubing a number makes it bigger ▪ The square of a number is always positive ▪ You can square root any number ▪ You can cube root any number

Algebra

Factorise quadratic expressions including the difference of two squares,	
$x^2 - 9 = (x + 3)(x - 3)$	Show me an example of a number which is can be written as the difference of two squares Show me an example of a two-term expression with a common factor of 2, -3, x etc…. True/Never/Sometimes: $(x + a)(x - a) = x^2 - a^2$ When will $(x + a)(x + b)$ have no ▪ x term ▪ positive x term ▪ negative x term ▪ positive constant?
Manipulate algebraic formulae, equations and expressions, finding common factors and multiplying two linear expressions	
Factorise the following expression: $m^4 – 2m^3 + 6m$ Expand the following, giving your answer in the simplest form possible: $(2b-3)^2$	Show me an example of a three term expression which has a common factor of: ▪ m^2 ▪ xy ▪ 2x2y True/Never/Sometimes: ax + b all squared is always greater than ax – b all squared when both a and b are any number between –10 and 10. Convince me that $(2x-3)^2 - (2x+3)^2 = -24x$
Derive and use more complex formulae and change the subject of a formula	
See the full range of examples on page 143 of the KS3 Framework supplement of examples. This includes examples such as: ▪ the area of a trapezium ▪ the area of an annulus ▪ the perimeter of a semicircle	(Given the variables and/or diagram) Show me a formula which could be used to find the: - surface area of this cylinder - area of this annulus What is wrong: $A = ((a+b)/2)h \equiv A-b = (a/2)h$ How can you change $A = ((a+b)/2)h$ so that (i) a is the subject (ii) b is the subject (iii) h is the subject? True/Never/Sometimes: You can change the subject of any formula Convince me that the area of an annulus is given by $A = \pi(R^2-r^2)$
Evaluate algebraic formulae, substituting fractions, decimals and negative numbers	
See the full range of examples on page 139 of the KS3 Framework supplement of examples. This includes examples such as: ▪ the volume of a sphere ▪ the volume of a torus	Show me an example of a formula that has the value 7 when a = -2 and b = -3 What is wrong: If $T = (5P^2)/(P+2)$ then: - T = 45 when P = -3 - T = 0.909090… when P = 0.2 Convince me that $x^2-3x+3 > 0$ for all values of x
Solve inequalities in two variables and find the solution set	
See the full range of examples on page 131 of the KS3 Framework supplement of examples. This includes examples such as: ▪ area bounded by three lines, two of which are parallel to the axes ▪ area bounded by a curve and a straight line	Show me an example of a coordinate pair that satisfies the inequalities ▪ $x < 5$ and $y > 2$ ▪ $y \geq x$ ▪ $2y < 3x - 2$ How can you change the inequalities that satisfy a region so that they satisfy a different region? Convince me that you need three linear inequalities to describe a region.

Sketch, identify and interpret graphs of linear, quadratic, cubic and reciprocal functions, and graphs that model real situations	
See the examples on pages 163, 171, 175 and 177 of the KS3 Framework supplement of examples.	Show me an example of an equation of a quadratic curve which does not touch the x-axis Show me an example of an equation of a parabola (quadratic curve) which ▪ is symmetrical about the y-axis ▪ is not symmetrical about the y-axis Show me an example of a function whose graph is not continuous (i.e. cannot be drawn without taking your pencil off the paper) True/Never/Sometimes: ▪ Cubic graphs have rotational symmetry ▪ Quadratic graphs have reflection symmetry in the y-axis What is the same/different about: $y=x^3$, $y=x^3+2x-4$ and $y=x^3+x^2-6x$
Understand the effect on a graph of addition of (or multiplication by) a constant	
Given the graph of $y=x^2$, use it to help sketch the graphs of $y=3x^2$ and $y=x^2+3$	Show me an example of an equation of a graph which moves (translates) the graph of $y=x^3$ vertically upwards (in the positive y-direction) What is the same/different about: $y=x^2$, $y=3x^2$, $y=3x^2+4$ and $\frac{1}{3}x^2$ True/Never/Sometimes: As 'a' increases the graph of $y=ax^2$ becomes steeper Convince me that the graph of $y=2x^2$ is a reflection of the graph of $y=-2x^2$ in the x-axis

Shape, Space and Measures

Understand and use congruence and mathematical similarity	
Use congruent triangles to prove that alternate angles are equal Understand and use the preservation of the ratio of side lengths in problems involving similar shapes (see p.191-193 of the KS3 Framework supplement of examples)	Show me and example of: ▪ Two congruent shapes ▪ Two similar shapes True/Never/Sometimes: ▪ Two right angled triangles are similar ▪ If you enlarge a shape you get two similar shapes ▪ All circles are similar Convince me that: ▪ Any two regular polygons with the same number of sides are similar ▪ Alternate angles are equal (using congruent triangles)
Understand and use trigonometrical relationships in right-angled triangles, and use these to solve problems, including those involving bearings	
Consider sine, cosine and tangent as ratios (link to similarity) Find missing sides in problems involving right-angled triangles in two dimensions Find missing angles in problems involving right-angled triangles in two dimensions	Show me and example of: ▪ A hypotenuse, opposite side, adjacent side ▪ A problem that can be solved using trigonometry ▪ A triangle in which the tangent of the angle is 1 ▪ A triangle in which the cosine is 0.5 What is the same/different about three triangles with sides 3, 4, 5 and 6, 8, 10 and 5, 12, 13 True/Never/Sometimes: You can use trigonometry to find the missing length/angle in triangles
Understand the difference between formulae for perimeter, area and volume in simple contexts by considering dimensions	
Identify which of the following expressions represent an area if 'a', 'b' and 'c' are lengths: ab+bc, 4abc, 5a+6b, 3ab² 2ab-c c(3b-2a)	Show me and example of: ▪ A formula for length/area/volume ▪ A possible formula for volume using the letters a, b and c as variables What is the same/different: ▪ two square metres, two hundred square centimetres and two metres squared ▪ pi times radius squared, pi times diameter, length times width, length times height, length times width times height True/Never/Sometimes: 10abc is a volume Convince me that 7ab + 3ac is an area

Handling Data

Estimate and find the median, quartiles and interquartile range for large data sets, including using a cumulative frequency diagram	
Estimate the median from a cumulative frequency curve Estimate the upper and lower quartiles from a cumulative frequency curve Find the interquartile range Use a cumulative frequency curve to find the number of pieces of data above / below a particular value	Show me an example of a set of data with a median of 10 and an interquartile range of 7 What is the same about/different about the two sets of data 7, 10, 8, 7, 4, 13, 9 and 7, 9, 3, 11, 9, 2, 6 True/Never/Sometimes: ▪ Lower quartile > Upper quartile ▪ Median = Minimum value ▪ Lower quartile < Upper quartile ▪ Interquartile range > Range ▪ Median = Lower quartile Convince me that the interquartile range for a set of data cannot be greater than the range
Compare two or more distributions and make inferences, using the shape of the distributions and measures of average and spread including median and quartiles	
Construct and interpret comparative box-plots See the range of examples on page 273 of the KS3 Framework supplement of examples	Show me an example of: ▪ A pair of box plots with the same median, but an interquartile range of one double the IQR of the other ▪ A box plot with negative skew ▪ An attribute / variable which has negative skew ▪ An attribute / variable which has positive skew What is the same about/different about the two sets of data 7, 10, 8, 7, 4, 13, 9 and 7, 9, 3, 11, 9, 2, 6 True/Never/Sometimes: ▪ Lower quartile > Upper quartile ▪ Median = Upper quartile ▪ Lower quartile = Upper quartile ▪ Interquartile range = Range ▪ Median < Lower quartile Convince me that (given two sets of data/box plots) you would choose to 'buy brand A' instead of 'brand B'
Know when to add or multiply two probabilities	
A bag contains 4 blue counters and 5 red counters. Billy picks a counter (without looking), replaces it, and then picks again. What is the probability that he picks one counter of each colour?	Show me an example of: ▪ A problem which could be solved by adding probabilities ▪ A problem which could be solved by multiplying probabilities
Use tree diagrams to calculate probabilities of combinations of independent events	
The probability that Nora fails her driving theory test on the first attempt is 0.1. The probability that she passes her practical test on the first attempt is 0.6. Complete a tree diagram based on this information and use it to find the probability that she passes both tests on the first attempt.	What is the same/different about the problems here: ▪ A bag contains 4 blue counters and 5 red counters. Julie picks a counter, replaces it, and then picks again. ▪ A bag contains 4 black counters and 5 pink counters. Sandra picks out two counters ▪ A bag contains 5 blue counters and 4 red counters. Walt picks a counter, replaces it, and then picks again.